**Section 1:**

**Q1: What is the expected number of rolls of a fair 6-sided die until you see two 5s in a row?**

**Main Concepts:**

1. Markov Chains: Analyzing states and transitions between states.
2. Expected Value: Calculating the average outcome over many trials.
3. Probability: Understanding probabilities associated with transitions between states.

**Q2: You continually roll a fair 10-sided die. What is the expected number of rolls until the lowest common multiple of all numbers that have appeared is greater than 2000?**

**Main Concepts:**

1. Probability: Determining the probabilities of different outcomes.
2. Number Theory: Understanding the concept of the lowest common multiple (LCM).
3. Expected Value: Estimating the average number of trials needed to reach a certain condition.

**Q3: Suppose the true value of V a car is uniformly distributed between 0 and 1000. You can bid any amount for the car, and if you bid the true value or more then you pay your bid and get the car. You know a very good salesman, and are confident in your ability to sell the car for 50% more than its true value. What should you bid to maximize your expected profit? Now suppose V ∼ Uniform[100, 1000], what should you bid?**

**Main Concepts:**

1. Expected Value: Calculating expected profit based on different bids.
2. Uniform Distribution: Understanding the properties and implications of a uniform distribution.
3. Decision Theory: Making optimal decisions under uncertainty.

**Section 2:**

**Q4: There are 50 ants on a 1m line. The 25 left-most ants are moving right and the 25 right-most ants are moving left. When 2 ants collide, they will both reverse direction. How many collisions will there have been in total once all ants have fallen off the end of the line?**

**Main Concepts:**

1. Discrete Mathematics: Analyzing movements and interactions of discrete entities.
2. Collision Theory: Understanding what happens when two objects meet.
3. Invariance Principle: Recognizing that the total number of collisions is independent of the interactions.

**Q5: Joe flips a fair coin 26 times, while Tim flips a fair coin 25 times. What is the probability that Joe gets strictly more heads than Tim?**

**Main Concepts:**

1. Probability: Calculating the likelihood of different outcomes.
2. Binomial Distribution: Understanding the distribution of a fixed number of independent trials.
3. Combinatorics: Counting the number of favorable outcomes.

**Q6: You are with your spouse at a party, along with 6 other couples. Some people shake hands at this party, although nobody shakes hands with themselves or their spouse. You observe that everyone (excluding yourself, but including your spouse) shook hands with a different number of people. How many people did your spouse shake hands with?**

**Main Concepts:**

1. Graph Theory: Analyzing relationships and connections between entities.
2. Combinatorics: Understanding the possible number of handshakes.
3. Problem Solving: Logical deduction to determine a unique solution based on given constraints.